

Rotation and Orientation of Soviet Sputnik III

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SOVIET Sputnik III carried a self-orienting magnetometer (1)¹ for measuring the intensity of the earth's magnetic field.

The magnetometer is designed with a movable frame, the normal to which, with the aid of special sensors and an automatic tracking system, coincides at any moment of time with the direction of the total magnetic field vector. The angles of rotation of the frame relative to the satellite body are measured by two transducers whose output is telemetered to the earth. Knowledge of the time dependence of these angles permits one to determine the parameters of motion of the satellite about the center of mass and its orientation in space.

The present paper proposes a method of solving this problem and describes the results of determining the parameters of rotation and orientation of the satellite for the interval from the first to the 109th orbit; the parameters of rotation are computed also from data of later orbits.

1. Method of Determining the Parameters of Orientation and Rotation of the Satellite

The authors will investigate the principle of determining the parameters of rotation and orientation of the satellite in accordance with the readings of the magnetometer transducer.

The motion of the satellite about the center of mass is affected by a number of perturbing factors: gravitational and aerodynamic moments (2-4), moments of electromagnetic drag (5), possible interaction of the magnetic moments of the circuits in the satellite itself with the earth's magnetic field, etc. Therefore, the motion of the satellite near the center of mass is a complex process. Under actual conditions the kinetic energy of satellite rotation considerably exceeds the work of external forces so that for a limited period of time (e.g., during a revolution of the satellite along its orbit) the effect of the perturbing forces is slight (2). Therefore, to a first approximation it can be assumed that during the specified limited period of time the motion of the satellite near the center of mass is the motion of a free solid body that is not affected by external forces. In such a case the motion of the satellite about the center of mass can be classified as the Euler-Poinsot motion. For Soviet Sputnik III, which has two equal major central moments of inertia, motion about the center of mass to the specified approximation is a regular precession. During this motion the satellite axis z' which coincides, as assumed, with the axis of the dynamic symmetry of the satellite, rotates uniformly at a constant angular velocity of precession $\dot{\psi}$ around vector \mathbf{L} of the satellite's kinetic moment (Fig. 1) which is stationary in absolute space. The angle of nutation ϑ between z' and \mathbf{L} is constant; in addition, the satellite rotates around its axis z' at a constant angular velocity of the characteristic rotation φ . Let X, Y, Z be the inertial coordinate system: axis Z is directed toward the earth's pole, axis X toward the vernal equinox; let ρ_0 be the angle between \mathbf{L} and axis Y , and let γ_0 be the angle between planes LY and XY (Fig. 1). The problem of determining the rotation and orientation of the satellite is reduced to the determination of parameters $\vartheta, \dot{\varphi}, \dot{\psi}, \rho_0, \gamma_0$ for every orbit and the establishment of a relationship between the position of the

satellite and time, that is, establishing at any moment of time the values of the angles of rotation and precession φ_0 and ψ_0 . At least one such relationship is necessary for every orbit. Parameters $\vartheta, \dot{\varphi}, \dot{\psi}, \rho_0, \gamma_0$, as assumed, are constant during a revolution of the center of mass of the satellite along the orbit but may change from revolution to revolution as a result of the continuous influence of small perturbing forces.

The readings of the magnetometer transducer permit one to determine all parameters $\vartheta, \dot{\varphi}, \dot{\psi}, \rho_0, \gamma_0, \varphi_0, \psi_0$ of rotation and orientation of the satellite.

Fig. 2 shows a diagram of the magnetometer gimbals. The axis of the outer frame coincides with the axis of the satellite; the outer frame can rotate around this axis. Its angle of rotation Δ is telemetered to the earth by means of the transducer, the readings of which are denoted by q_1 . Angle Δ is reckoned from a certain fixed axis x' in the satellite, perpendicular to the symmetry axis z' of the satellite. Axis x' is normal to the outer frame when $\Delta = 0$.

The axis of the inner gimbal is perpendicular to the axis of the outer gimbal. The normal to the inner gimbal is always set in the direction of the vector of magnetic intensity \mathbf{H} by rotating the outer and inner gimbals by the requisite angles relative to the satellite body. The angle of rotation of the inner gimbal is telemetered to the earth by means of a transducer, the readings of which are denoted by q_2 . A particular feature of the design of the magnetometer carried on Sputnik III is that q_2 is dependent on q_1 (there is no reciprocal relationship); namely, owing to a special system of gears in the orientation block, one third of the transducer reading q_1 is added to the transducer reading q_2 . Hence, the independent part of reading q_2 equals

$$\lambda \sim q_2 - \frac{1}{3}q_1$$

Angle $q_2 - \frac{1}{3}q_1$, with an accuracy to the additive constant, determining zero of λ , is the angle between the symmetry axis of the satellite z' and the magnetic intensity vector \mathbf{H} (Fig. 2). Angle Δ is the angle between the plane of reckoning of angle λ (plane $z'H$) and a fixed plane in the satellite passing through axis z' . Thus, angles λ and Δ completely determine the position of the satellite relative to the magnetic line of force. These angles change in time as a result of the rotation of the satellite about the center of mass, the motion of the center of mass itself along the orbit, and the change of direction of the magnetic intensity vector \mathbf{H} , related to this motion.

Let the position of \mathbf{H} be given by the two coordinates ρ and γ , determined analogously by coordinates ρ_0 and γ_0 (Fig. 1). As the orbit of the satellite is known and the earth's magnetic field also can be considered sufficiently known for the purpose of determining the satellite's orientation, then ρ and γ are known as functions of time. In such a case the time dependence of $\lambda(t)$ and $\Delta(t)$ is determined by the dependence of λ and Δ on the parameters of orientation and rotation of the satellite. Consequently, the parameters of rotation and orientation are found from the telemetry data which determine the dependence of $\lambda(t)$ and $\Delta(t)$.

Magnetic coordinates ρ and γ are computed by means of Vestine's tables for magnetic inclination D and magnetic inclination I given in terms of functions of the geographic coordinates and altitude over the earth's surface. Tables of I and D for altitudes $h = 100, 300, 500, 1000, 5000$ km were used. The I and D tables are set up for every altitude at intervals of 10° in latitude φ and at intervals of 30° in longitude λ . As a result, the determination of I and D at any required point

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¹ Numbers in parentheses indicate References at end of paper.

$h_0, \bar{\varphi}_0, \bar{\lambda}_0$ involves triple interpolation. Interpolation is carried out over $\bar{\varphi}, \bar{\lambda}, h$, in that order; it is possible to limit the interpolation to the linear one.

Let $\cos(\mathbf{H}\hat{X}), \cos(\mathbf{H}\hat{Y}), \cos(\mathbf{H}\hat{Z})$ be the direction cosines of vector \mathbf{H} of magnetic intensity with respect to the inertial axes X, Y, Z . Then the sought coordinates ρ and γ are determined as follows:

$$\cos \rho = \cos(\mathbf{H}\hat{Z}) \quad \tan \gamma = \cos(\mathbf{H}\hat{Z}) / \cos(\mathbf{H}\hat{X}) \quad [1.1]$$

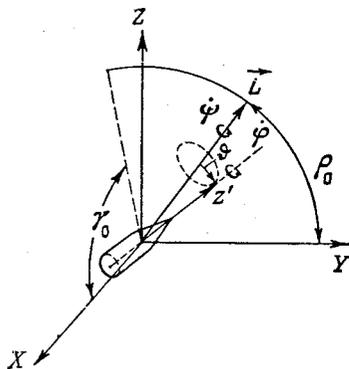
Angle ρ is defined in the first and second quadrants; the quadrant which contains angle γ is dependent on the circumstance that the sign of $\sin \gamma$ coincides with the sign of $\cos(\mathbf{H}\hat{Z})$ and the sign of $\cos \gamma$ with that of $\cos(\mathbf{H}\hat{X})$.

The authors now introduce absolute longitude λ^* , reckoned from the vernal equinox (that is, from axis X):

$$\lambda^* = t_{zv}^{(sec)}(360^\circ/86,400) + (t_M - 3^h)^{(sec)} \cdot (360^\circ/86,164.09) + \bar{\lambda}$$

Here $t_{zv}^{(sec)}$ is the longitude of the Greenwich meridian, reckoned from the vernal equinox at Greenwich midnight and expressed in seconds of time (taken from the astronomical almanac for every day of the year); t_M is the Moscow standard time at the point under investigation; $(t_M - 3^h)^{(sec)}$ is Greenwich time (in seconds) at the point under investigation; constant 86,164.09 determines the duration (in seconds) of the sidereal day.

Fig. 1 Parameters of orientation and rotation of the satellite



Then the values of $\cos(\mathbf{H}\hat{X}), \cos(\mathbf{H}\hat{Y}), \cos(\mathbf{H}\hat{Z})$ are determined as follows:

$$\begin{aligned} \cos(\mathbf{H}\hat{X}) &= -\sin \varphi \cos \lambda^* \cos I \cos D - \sin \lambda^* \cos I \sin D - \cos \varphi \cos \lambda^* \sin I \\ \cos(\mathbf{H}\hat{Y}) &= -\sin \varphi \sin \lambda^* \cos I \cos D + \cos \lambda^* \cos I \sin D - \cos \varphi \sin \lambda^* \sin I \\ \cos(\mathbf{H}\hat{Z}) &= \cos \varphi \cos I \cos D - \sin \varphi \sin I \end{aligned} \quad [1.2]$$

Next we study the dependence of the reading of the orientation transducers on the parameters of motion of the satellite.

As pointed out previously, it can be assumed that during a short period of time (of the order of one revolution of the satellite along the orbit) motion of the satellite about the center of mass is a regular precession. Let $\mu \dot{\psi} \equiv \text{const}$ be the angular velocity of precession, $\dot{\varphi} \equiv n = \text{const}$ be the angular velocity of characteristic rotation, $\vartheta = \text{const}$ be the angle of nutation.

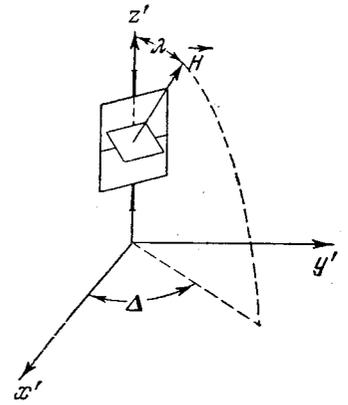
Then, as already known

$$[(A/C) - 1] \mu \cos \vartheta = n \quad [1.3]$$

where A and C , respectively, are the transverse and longitudinal principal central moments of inertia of the satellite.

Let us investigate the unit sphere associated with the center of mass of the satellite. Inertial axes X, Y intersect this sphere at points X and Y , the vector of the kinetic moment, in point L (Fig. 3); $LY = \rho_0, \angle LYX = \gamma_0$; γ_0 is considered positive for clockwise reckoning about axis Y . The trace of the z' axis of the satellite in regular precession is

Fig. 2 Diagram of magnetometer gimbals



a circle of radius ϑ on the unit sphere with the center in L . The angle between arcs LY and Lz' , reckoned counterclockwise around L , is denoted by $\tilde{\psi}$; hence $\tilde{\psi} = \mu$.

Let H be the trace of the vector of magnetic intensity on the unit sphere, and ρ and γ be its coordinates. Then $z'H = \lambda$ is characterized, as indicated, by transducer readings q_1 and q_2 :

$$\lambda = q_1 - \frac{1}{3}q_2 + a$$

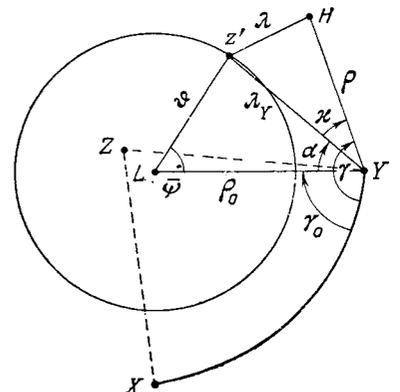
where a is constant, determining zero value of λ .

We shall now derive the time dependence of λ . Let $\lambda_y = z'Y$; κ be the angle, reckoned clockwise around Y from arc λ_y to arc $YH = \rho$; α be the angle, reckoned clockwise around Y from arc $LY = \rho$ to arc $z'Y = \lambda_y$. Then, the system of spherical triangles $Lz'Y$ and $z'HY$ gives

$$\begin{aligned} \cos \lambda &= \cos \rho \cos \lambda_y + \sin \rho \sin \lambda_y \cos \kappa \\ \cos \lambda_y &= \cos \vartheta \cos \rho_0 + \sin \vartheta \sin \rho_0 \cos \tilde{\psi} \\ \cos \alpha &= \frac{\cos \vartheta - \cos \rho_0 \cos \lambda_y}{\sin \rho_0 \sin \lambda_y} \\ \cot \alpha &= \frac{\sin \rho_0 \cot \vartheta - \cos \rho_0 \cos \tilde{\psi}}{\sin \tilde{\psi}} \end{aligned} \quad [1.4]$$

Eqs. [1.4] completely determine λ as a function of time and the parameters of precession and orientation. Actually, $0 \leq \lambda \leq 180^\circ$; therefore, the formula for $\cos \lambda$ is sufficient for a unique determination of Z . In this formula, coordinates $\rho(t)$ and $\gamma(t)$ of the vector of magnetic intensity are known as functions of time in agreement with [1.1] and [1.2]; angle $\lambda_y, (0 \leq \lambda_y \leq 180^\circ)$ is determined through ϑ, ρ_0 , and $\tilde{\psi} = \tilde{\psi}_0 + \mu t$; for a unique determination of angle α contained in the formula for $\cos \lambda$, the formulas for $\cot \alpha$ and $\cos \alpha$ dependent, as can be seen on $\rho_0, \tilde{\psi} = \tilde{\psi}_0 + \mu t, \vartheta$ are sufficient.

Fig. 3 Orientation parameters on the unit sphere surface



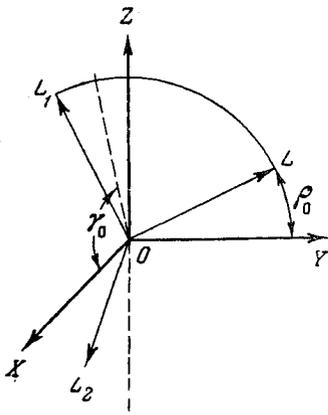


Fig. 4 Auxiliary system of coordinates

Thus, Eqs. [1.4] give the sought dependence

$$\lambda = \lambda(t, \rho_0, \gamma_0, \vartheta, \mu, \bar{\psi}_0)$$

The parameters of characteristic rotation are not contained in this formula, because λ describes only the position of the satellite axis. The other telemeter angle, namely Δ , is dependent on the characteristic rotation. In order to determine this dependence, the system of coordinates L, L_1, L_2 should be introduced; this system complements the vector of kinetic moment L to form the right-hand triplet so that axis L_1 lies in plane LOY , while $\angle L_1Y \leq 90^\circ$, so that when $\rho_0 = 90^\circ$, axis L_1 coincides with the negative axis Y , then axis L_2 lies in plane XZ (Fig. 4). Couple a system x', y', z' with the satellite; here z' is the symmetry axis of the satellite and x' lies in the "zero" plane q_1 (Fig. 2). Now introduce the standard Eulerian angles φ, ψ, ϑ (Fig. 5) in system L, L_1, L_2 , so that the angle of precession ψ is the angle reckoned from OL_1 to the line of nodes OC , and the angle of characteristic rotation φ is reckoned from OC to Ox' .

Fig. 6 shows the surface of the unit sphere with the representation of the angles and traces of the introduced axes. A study of this figure indicates that the following relationship exists between the Eulerian angle of precession ψ and the previously introduced angle $\bar{\psi}$:

$$\psi = \bar{\psi} - 90^\circ \tag{1.5}$$

and

$$\Delta = \varphi + \nu \tag{1.6}$$

where ν is the angle between planes $z'H$ (H is the trace of the vector of magnetic intensity) and $z'C$. The dependence of ν on the parameters of motion is not written out here because it is not required in the following; it is only pointed out that ν is independent of φ :

$$\nu = \nu(\rho_0, \gamma_0, \gamma, \bar{\psi}_0 + \mu t)$$

Thus, angle Δ , in agreement with [1.6], depends linearly on φ so that it is possible to determine, on the basis of $q_1 \sim \Delta$, the angular velocity of the characteristic rotation $\dot{\varphi}$ by segregating

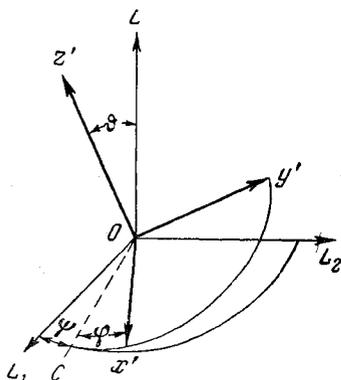


Fig. 5 Euler angles relative to the auxiliary coordinate system

the linear component in this written formula, inasmuch as ν is periodically dependent on $\rho, \gamma, \bar{\psi}_0 + \mu t$. It should be noted also that when point H lies on arc C_1z' (Fig. 6), then $\lambda = \lambda_{extrp}, \nu = \pm 90^\circ$; therefore

$$\varphi(\lambda = \lambda_{extrp}) = \Delta(\lambda = Z_{extrp}) \pm 90^\circ \tag{1.7}$$

This formula makes it possible to determine the time dependence of quantity φ . The sign to be selected in [1.7] is always uniquely determined from supplementary considerations depending on the mutual arrangement of the three points H, z', L .

The time dependence of angles $\lambda(t)$ and $\Delta(t)$, determined from the measurements of the magnetometer transducers, is in the form of certain oscillating curves given by expressions [1.4] and [1.6]. The parameters of precessional motion and the orientation of the vector of the kinetic moment must be determined from these curves.

Let us investigate $\lambda(t)$ (the angle between vectors H and z'). Fig. 3 or 6 shows that if vector H would be fixed, then angle λ during the half period of precession (half rotation of axis z' around axis L) would change from the minimum to the maximum value, and hence the oscillation cycle λ would coincide with the precession period T_ψ . Actually, vector H moves slowly and describes on the unit sphere a loop of its trajectory during the half period of rotation of the satellite along the orbit. Consequently, curve $\lambda(t)$ is the result of the interaction of two kinds of oscillation motion: a short-period motion brought about by the precession of the satellite and a long-period motion caused by the movement of the vector of magnetic intensity. Calculation of the number of short-period oscillations during the period of time which contains an integral number of these oscillations suffices for an approximate determination of precession period T_ψ . The accuracy of determining T_ψ is the higher, the smaller this cycle is, compared with the half period of revolution of the satellite along the orbit. The inaccuracy in the determination of T_ψ is dependent on the movement of the magnetic intensity vector H and, inasmuch as this movement is known, after all parameters of orientation and precession have been determined to a first approximation, it is possible to compute the correction to the first approximation of T_ψ and determine T_ψ more accurately. The angular velocity of precession $\dot{\psi} \equiv \mu$ is determined, after this, by $\mu = 360^\circ/T_\psi$. For Soviet Sputnik III the corrections to the first approximation T_ψ were small (1-2%) and were determined, practically, by comparing the magnetometer readings with the readings of other devices (see Sec. 3).

The magnitude of the angular velocity of the satellite's characteristic rotation is determined in the same simple manner. In agreement with [1.6] and [1.7], angular velocity $\dot{\varphi}$ of the satellite's characteristic rotation is determined by the slope of the linear component $q_1[\Delta(t)]$:

$$n \equiv \dot{\varphi} = \dot{\Delta}$$

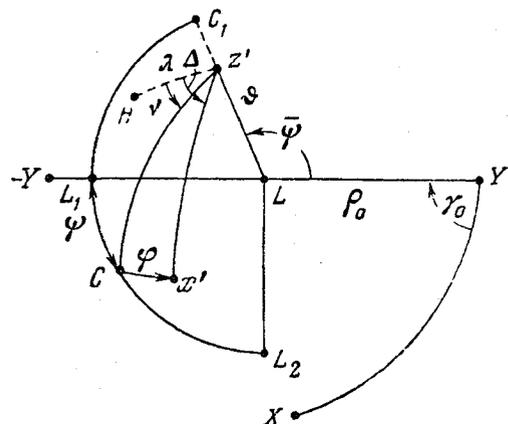


Fig. 6 Euler angles on the surface of a unit sphere

(here L_Δ is the linear part of Δ). The third parameter (angle of nutation ϑ) is found when n and μ from [1.3] are known.

The parameters of orientation ρ_0 and γ_0 were determined by the method of direct matching of the parameters in [1.4] for the purpose of obtaining maximum matching of the theoretical curve $\lambda(t)$, derived in this manner, with the readings of the magnetometer transducer already available.

An algorithm of successive approximations of the parameters $\rho_0, \gamma_0, \psi_0, \vartheta, \mu$, was used giving the best fit between the theoretical and the experimental curves in the sense of the method of least squares (6), that is, such a successive selection of the specified parameters that

$$\sum_i [\lambda(t_i) - \lambda_i]^2 = \min$$

where $\lambda(t_i)$ is the theoretical values of λ at the moments of time t_i , and λ_i is the experimental values at the same moments of time. Mathematically, the problem is reduced to solving a system of algebraic linear inhomogeneous equations, the coefficients of which vary from approximation to approximation. Values of parameters derived from some other considerations can be used, or a selection can be made from a whole series of parameter values for the first approximation on an orbit. The result of the calculation for this orbit can be selected as the first approximation for the next orbit.

The specified algorithm of the method of least squares was programmed on a high speed electronic computer. The results of the computation and their analysis are given in the following section.

2. Results of the Processing of the Experimental Data

Sec. 1 indicates that the parameters of precessional motion can be determined in two ways. Either find ϑ, μ by selecting the constants, contained in the equation for $\lambda(t)$, along with the coordinates ρ_0, γ_0, ψ_0 , or determine ϑ and μ directly from the reading q_1 and q_2 and then select only ρ_0, γ_0, ψ_0 .

Both of these methods have been used. The criterion of correctness of the computation is the coincidence of the curve obtained from the final values of the sought parameters with the experimental curve obtained by telemetering.

Fig. 7 shows an example of such a coincidence of the calculated curve $\lambda(t)$ (dashed line with small circles) and the experimental curve (solid line). It is seen readily that the coincidence is completely satisfactory. The same kind of coincidence of the calculated curve with the experimental one has been obtained for all orbits for which the data have been processed.

It is pointed out that calculation requires knowledge of the zero approximation from which the calculation starts. Quantities n, μ , and, as a consequence, ϑ are determined with sufficient accuracy directly from the readings q_1 and q_2 . The zero approximation ψ_0 was determined from a series of trials. It was possible to determine the zero approximation of ρ_0 and γ_0 on the 56th orbit. As pointed out, the values of angle $\lambda(t)$ cannot extend beyond the interval $[0, \pi]$. If the rotation of

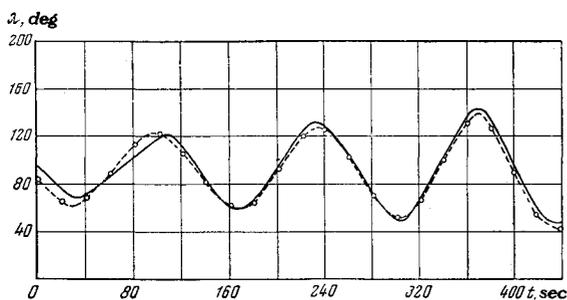


Fig. 7 Coincidence of experimental and theoretical curves (orbit 15)

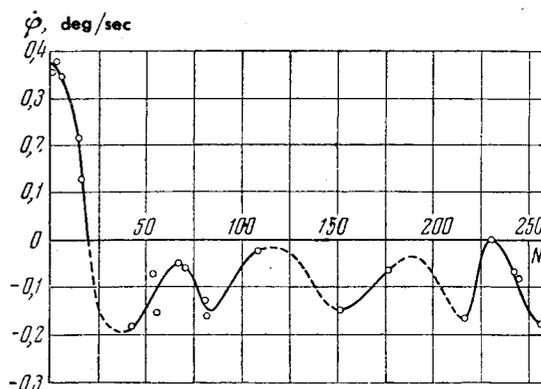


Fig. 8 Angular velocity of the characteristic rotation of the satellite. Dashed line denotes curve sections obtained by extrapolation

the satellite is such that angle λ tends to extend beyond these limits, then the outer gimbal of the magnetometer rotates around the longitudinal axis and angle $\lambda(t)$ remains within the limits $[0, \pi]$. At the instant of rotation, angle $\lambda(t)$ is close to either 0 or π ; this means that, if the magnitude of the angle of nutation is known, it is possible to determine the locus of the points on the unit sphere of the trace of the vector of the kinetic moment. The appropriate zero approximation for the 56th orbit is found by selecting the values of the zero approximations from the family of these points. After the final values of ρ_0 and γ_0 have been determined for the 56th orbit they are used as the zero approximation for the neighboring orbit.

The existing programs show that the calculation is adequately stable for a variation of the zero approximation; that is, it gives the same results for essentially different zero approximations. As already stated, the calculations were made in accordance with two programs; three parameters were selected for the first, five for the second. The results of the two programs are in good agreement (see the Tables). The difference of the angular coordinates for the two methods does not exceed 10–15°. An exception is parameter ψ_0 (initial definition of the position satellite's axis). The values of this parameter, obtained by two methods, in separate cases differ by 30–40°. This means that the computation of the orientation of the devices requires an additional correction for parameter ψ_0 . The variation of ψ_0 , in principle, leads only to a shift of the maximum of the reading of the devices; therefore, when such readings are available, it is simple to select such ψ_0 that the calculated orientation matches the reading of the corresponding device. It is shown in Sec. 3 that for a matching of this type a variation of ψ_0 within the limits of 30° from the initial value actually suffices.

Let us now study the results of the processing of the data. The processing of the data recorded by the magnetometer transducer in accordance with the methods previously specified gave the following results.

1. Angular Velocity of Characteristic Rotation $\dot{\varphi} \equiv n$

The values of this parameter (in angular deg/sec) for every orbit are compiled in Table 1. Here N is the number of orbits. Function $\dot{\varphi}(N)$ is depicted in Fig. 8.

It can be seen readily that the angular velocity of rotation has been reduced from 0.375 grad·sec⁻¹ on the third orbit to zero in the interval between orbits 17 and 42. On orbit 42 $\dot{\varphi} = -0.184$ grad·sec⁻¹, that is, the rotation of the satellite around the symmetry axis has reversed its direction. From then on $\dot{\varphi}$ fluctuates around the average value $\dot{\varphi}_{av} \approx -0.1$ grad·sec⁻¹ within the limits $0 \geq \dot{\varphi} > -0.2$ grad·sec⁻¹.

Such behavior of the angular velocity probably can be explained by the interaction of currents in the electrical wiring

Table 1

N	1	3	15	17	42	54
ϕ	0.3575	0.375	0.2133	0.125	-0.184	-0.076
N	56	68	70	81	82	109
ϕ	-0.153	-0.049	-0.056	-0.131	-0.16	-0.026
N	161	176	216	230	243	257
ϕ	-0.15	-0.07	-0.17	0	-0.07	-0.18

system of the satellite with the earth's magnetic field. Actually, the magnetic field produced by currents on the satellite will interact in various ways with earth's magnetic field, either increasing or reducing the velocity of rotation of the satellite. Were the currents symmetrical and continuous, then the overall effect would equal zero; however, since the currents are not symmetrical (only the solar batteries facing the sun are working) and not continuous, then it is possible that either the satellite is decelerated or its rotation is accelerated, depending on the position of the satellite relative to the earth's magnetic field. It has been determined that for Sputnik III such an effect can be sufficiently strong and that it decelerates the rotation of the satellite on the first orbits as shown in Table 1.

It is possible that the interaction of Foucault currents in the satellite skin with the earth's magnetic field also has a certain role. However, as shown in Ref. 5, the role of this effect is slight in the case of heavy satellites and cannot explain the observed deceleration. The same can be said about atmospheric friction. Other factors (variation of the moments of inertia during the opening and closing of the louvers, collisions with micrometeorites, etc.) have a negligible effect on the variation of the angular velocity ϕ .

2. Period of Precession

The values of the precession period T_ψ on every orbit are compiled in Table 2. These periods are obtained directly from the readings of the magnetometer transducer. These periods must be changed by 2 to 5 sec in order to match them with the reading of other devices.

Table 2 indicates that the period of precession increases from 135-136 sec on the first orbits to 195 sec on 283rd orbit. Fig. 9 shows the function $T_\psi(X)$. The increase of the period of precession, that is, the decrease of the angular velocity ψ of precession, can be explained in the same way as the decrease of angular velocity ψ of the characteristic rotation—namely, by the interaction of the currents in the satellite with the earth's magnetic field. However, in this case the influence of this effect is rather slight because the initial angular velocity of precession is many times greater than the initial angular velocity of characteristic rotation (on the first loop $\psi = 2.55 \text{ grad}\cdot\text{sec}^{-1}$ and $\phi = 0.357 \text{ grad}\cdot\text{sec}^{-1}$). A certain nonuniformity of the increase of the period of precession can be explained not only by inaccurate processing of the data but probably reflects the actual fluctuations of the angular velocity of precession, analogous to the fluctuations of the angular velocity of the characteristic rotation.

Table 3 shows (in angular degrees per second) the precession rates which are used as the constant parameter in method I of determining the parameters of orientation ρ_0 , γ_0 , and the

angular velocities of precession obtained along with ρ_0 , γ_0 , in method II where five parameters were being estimated.

The greatest difference between the two parts of Table 3 ($\sim 0.05 - 0.06 \text{ grad}\cdot\text{sec}^{-1}$) is characterized by the precession of determining the angular velocity of precession. This corresponds to an error of 3 to 6 sec in determining the precession period.

3. Angle of Nutation

For Sputnik III the ratio of transverse to longitudinal moments of inertia amounts to

$$A/C \approx 2.5$$

Knowing A/C , ϕ , ψ , it is possible to compute the angle of nutation ϑ according to [1.3]. It appears that the angle of nutation is close to 90° , deviating from this value by not more than 6° on individual orbits (Table 4). This means that the satellite "tumbles."

The investigated parameters T_ψ , ϕ , ϑ completely describe the system of rotation of the satellite about the vector of the kinetic moment. Let us now study the results of the determination of the orientation of the vector of the kinetic moment.

4. Orientation of the Vector of the Kinetic Moment

As has been specified, coordinates ρ_0 and γ_0 of the vector of the kinetic moment were determined by two methods: the method with three varying parameters (I) and the method with five varying parameters (II). The results obtained in parallel by the two methods are compiled in Table 5.

Table 5 shows that the agreement between methods I and II is satisfactory. The maximum difference (in one case) is approximately 15° ; in other instances the difference amounts to $5^\circ-10^\circ$, which characterizes the accuracy of determining the vector orientation of the kinetic moment. The difference between the coordinates on separate orbits is explained by the slow motion of the vector of the kinetic moment in space as a consequence of the influence of perturbing factors (2,4).

Fig. 10 depicts the dependence of ρ_0 and γ_0 on N for the two computing methods (I and II). It can be seen readily that all of these points satisfactorily coincide with the solid curve. Orbits 5, 68, and 82 are an exception. Points ρ_0 and γ_0 do not coincide with the curve for these orbits. Fig. 10 also shows curves which describe functions $\rho_0(N)$ and $\gamma_0(N)$ with an accuracy to $5^\circ-10^\circ$ (without the points of orbits 5, 68, 82).

Table 2

N	T_ψ , sec						
1	136	54	159	109	171	245	185
2	135	56	157	161	170	256	190
3	136	68	162	176	185	257	195
15	136	70	170	216	185	258	195
17	138	81	167	230	182	260	195
42	144	82	156	243	183	283	195

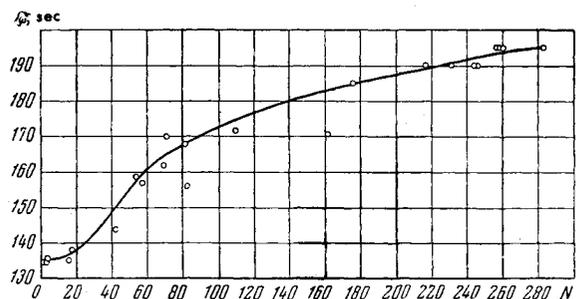


Fig. 9 Period of precession of the satellite

Table 3

<i>N</i>	1	3	5	15	42	54
I) ψ	2.55	2.52	2.57	2.56	2.50	2.30
II) ψ	2.52	2.52	2.61	2.51	2.35	2.29
<i>N</i>	56	68	70	81	82	109
I) ψ	2.30	2.14	2.18	2.14	...	2.14
II) ψ	...	2.20	2.24	2.175	2.13	2.11

Table 4

<i>N</i>	1	3	5	15	17	42
ϑ	84°.5	84°.5	84°	87°	88°	87°
<i>N</i>	54	56	68	70	81	109
ϑ	87°	87°	88°	87°	87°	88°

The results obtained for the initial angle of precession $\bar{\psi}_0$ by computing in accordance with the two programs are described briefly. As already pointed out, this parameter is determined less precisely; this is clearly seen in Table 6, which also shows that the computation according to the two programs is in good agreement (orbit 42) and that the results for orbit 15 differ by 40°. Therefore, a more precise value of $\bar{\psi}$ is obtained by comparing the readings of other devices. Such a correction can be made readily because the positions of the maxima of the readings are dependent on $\bar{\psi}_0$ (see Sec. 3).

It is of interest to determine the type of motion performed by the vector of the kinetic moment relative to the satellite orbit. For this purpose, the absolute angular coordinates ρ_0 , γ_0 are converted into orbital coordinates related to the perigee of the orbit (Fig. 11); the values of ρ_0 and γ_0 computed by method I (Table 5, I) are selected.

5. Vector Orientation of the Kinetic Moment Relative to the Orbit

Let axis \bar{X} of the orbital system of coordinates, related to the perigee, be directed along the velocity vector at the perigee,

Table 5

<i>N</i>	I		II	
	ρ_0	γ_0	ρ_0	γ_0
1	42°	74°
3	40	90	36°	93°
5	71	98	82	101
15	40	116	34	132
42	50	135	49	133
54	62	150	64	149
56	69	150
68	37	166	49	154
70	67	155	78	146
81	76	148	80	140
82	54	151	52	155
109	75	146	66	150

Table 6

<i>N</i>	3	15	42
I) $\bar{\psi}_0$	170°	252°	352°
II) $\bar{\psi}_0$	201°	292°	354°

Table 7

	<i>Y</i>	<i>Z</i>	<i>X</i>
\bar{X}	<i>a</i>	<i>b</i>	<i>c</i>
\bar{Y}	<i>a</i> ₁	<i>b</i> ₁	<i>c</i> ₁
\bar{Z}	<i>a</i> ₂	<i>b</i> ₂	<i>c</i> ₂

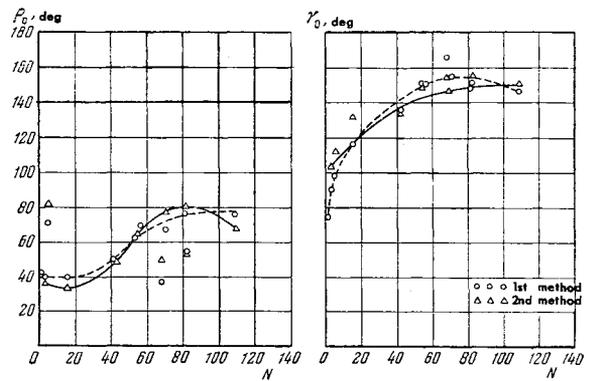


Fig. 10 Orientation of the vector of the kinetic moment—dotted line (method I), solid curve (method II)

axis \bar{Y} along the normal to the orbital plane, and axis \bar{Z} along the radius-vector at the perigee. The position of the vector of kinetic moment \mathbf{L} relative to the orbital system is given in terms of angle Θ between \mathbf{L} and axis \bar{X} and angle λ between the orbital plane $\bar{Z}\bar{X}$ and plane $\mathbf{L}\bar{X}$ (Fig. 11).

Let i be the inclination of the orbit to the earth's equator, Ω the longitude of the ascending node of the orbit from the point of vernal equinox, and ω the angle between radius-vectors at the perigee and the ascending node. Then the position of system $\bar{X}, \bar{Y}, \bar{Z}$ relative to the inertial system X, Y, Z is given in the table of direction cosines (Table 7).

$$\begin{aligned}
 a &= -\sin\omega \sin\Omega + \cos\omega \cos\Omega \cos i \\
 b &= \cos\omega \sin i \\
 c &= \sin\omega \cos\Omega - \cos\omega \sin\Omega \cos i \\
 a_1 &= -\cos\Omega \sin i \\
 b_1 &= \cos i \\
 c_1 &= \sin\Omega \sin i \\
 a_2 &= \cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i \\
 b_2 &= \sin\omega \sin i \\
 c_2 &= \cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i
 \end{aligned}$$

Let A_1, A_2, A_3 be the direction cosines of vector \mathbf{L} of the kinetic moment with respect to the inertial axis X, Y, Z , respectively:

$$\begin{aligned}
 A_1 &= \cos\rho_0 & A_2 &= \sin\rho_0 \sin\gamma_0 \\
 A_3 &= \sin\rho_0 \cos\gamma_0
 \end{aligned}$$

Then the direction cosines $\alpha_0, \alpha_1, \alpha_2$ of vector \mathbf{L} with perigee axes $\bar{X}, \bar{Y}, \bar{Z}$, respectively, are expressed by

$$\begin{aligned}
 \alpha_0 &= A_1 a + A_2 b + A_3 c \\
 \alpha_1 &= A_1 a_1 + A_2 b_1 + A_3 c_1 \\
 \alpha_2 &= A_1 a_2 + A_2 b_2 + A_3 c_2
 \end{aligned}$$

whence

$$\cos\Theta = \alpha_0 \quad \tan\lambda_0 = \alpha_1/\alpha_2$$

while the quadrant which contains λ is determined by the sign

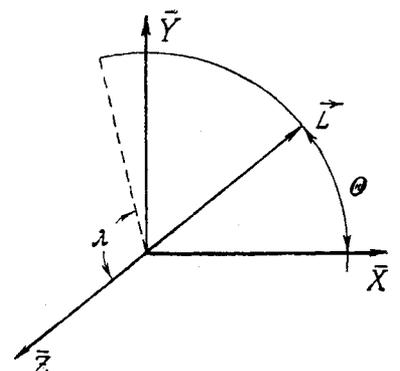


Fig. 11 Orbital coordinate system

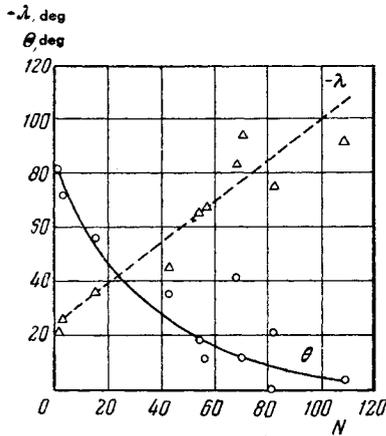


Fig. 12 Orientation of the vector of the kinetic moment relative to the orbit

of α_1 which coincides with the sign of $\sin\lambda$ and the sign α_2 which coincides with the sign of $\cos\lambda$. Elements Ω, i, ω , for every orbit are given in the tables of elements of Sputnik III.

The results of the calculations according to these formulas are shown in Figs. 12 and 13. It can be seen that the variation of λ is uniform at a rate of $\sim 0.76^\circ/\text{orbit}$. Angle θ between the vector of the kinetic moment, and the perigee tangent varies monotonically from $\sim 80^\circ$ on the first orbit to values close to zero on the 100–110th orbit. The rate of variation of θ on the first 10–20 orbits is $\sim 1.5^\circ/\text{orbit}$, after that the rate of variation of θ decreases to a value close to zero. Hence, vector L moves in such a manner that toward the end of the time interval under investigation (from the first to the 109th orbit) vector L tends to coincide with the direction of the perigee tangent. Fig. 13 depicts the motion of vector L in polar coordinates $\theta\lambda$; the point of origin of the coordinates is the trace of the vector of velocity of the center of mass of the satellite at perigee. The curve in Fig. 13 is plotted in accordance with the curves in Fig. 12 describing the average motion of the vector of the kinetic moment. The deviation of separate points which correspond to experimental data does not exceed 10° .

Since the angle of nutation ϑ between the satellite axis and vector L of the kinetic moment is close to 90° and the direction of vector L on the last orbits is close to the direction of the velocity vector at the perigee, then, on these orbits, the satellite passes the perigee in a regime of maximum aerodynamic drag resulting in a decrease of the lifetime of the satellite.

Comparison with the theoretical investigation of the secular motion of the vector of the kinetic moment under the influence of aerodynamic and gravitational perturbations (4) shows that the velocities of secular motion and the nature of the curve in Fig. 13, within the limits of accuracy, are in good agreement with theoretical results. Among the classes of trajectories of the tip of the vector of the kinetic moment on

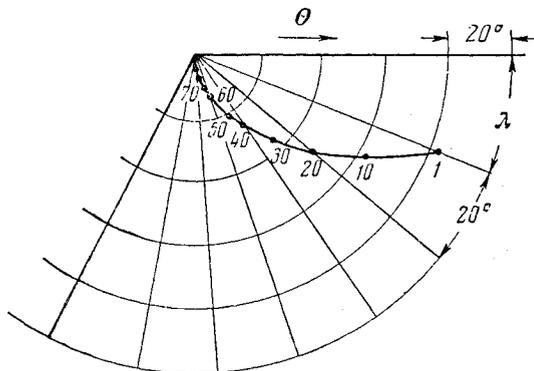


Fig. 13 Trajectory of the vector of the kinetic moment relative to the orbit

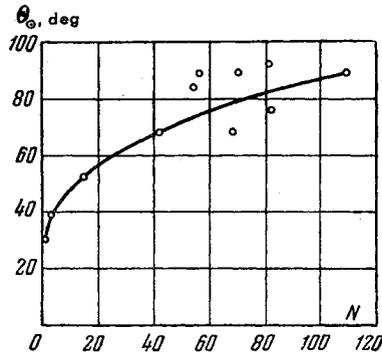


Fig. 14 Orientation of the vector of the kinetic moment relative to the sun

the unit sphere, specified in Ref. 4, there are two classes ("tripolar" and "penta-polar") which contain special trajectories asymptotically approaching the origin of the coordinates, that is, they are close to the trajectories in Fig. 13.

In agreement with Ref. 4, any trajectory which is different from the special one and is included in a certain vicinity of the origin of the coordinates, again departs from this region after a certain period of time (pole of unstable equilibrium). It is not known whether this was the case for Sputnik III on subsequent orbits because the existing theory (2,4) does not take electromagnetic effects and other possible factors into account which could explain the observed deceleration of the rotation velocity of the satellite. According to Ref. 4, in addition to the position of unstable equilibrium of vector L , there also is a pole of stable equilibrium. When such small dissipative forces are effective, such as the forces of electromagnetic drag and the forces of aerodynamic friction, this pole can be transformed into the pole of asymptotic stability. The curve in Fig. 13 can be discussed in the same manner, either as motion in the vicinity of the position of unstable equilibrium of vector L or as motion in the vicinity of stable equilibrium distorted by the influence of dissipative forces. Which of these instances actually occurs can be determined only by processing the data obtained for a larger number of orbits and with the aid of a more general theory of the motion of vector L . The real picture of the motion of vector L does not coincide with the theoretical picture in Refs. 2 and 4 because parameters ϑ and ψ are variable (these papers assume that they are stable). Such a difference can be brought about also by a possible additional motion of vector L under the influence of electromagnetic forces. The effect of the variation of ϑ and ψ can be computed readily. The problem of how the electromagnetic effects directly influence the motion of vector L requires additional investigation. If this effect is small, then,

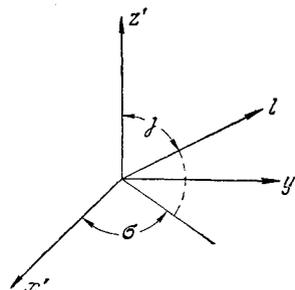


Fig. 15 Angles determining the attitude of a given axis in the satellite relative to the satellite

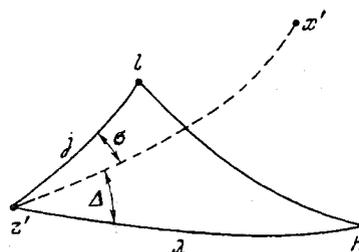


Fig. 16 Computation of the orientation relative to the earth's magnetic field

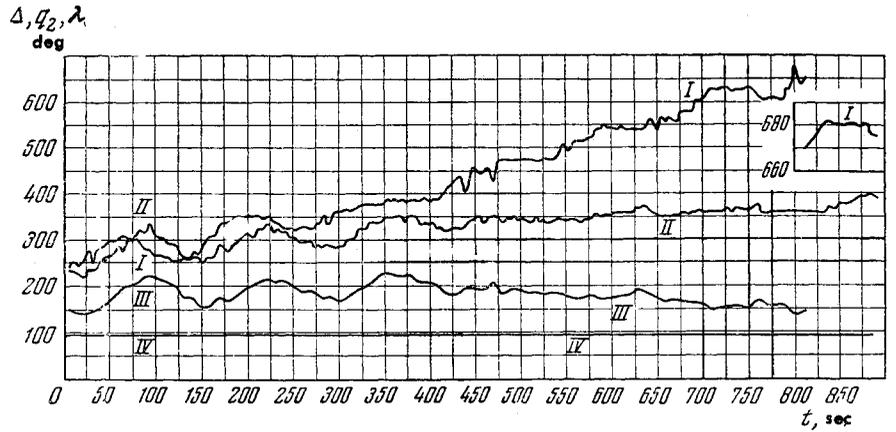


Fig. 17 Magnetometer reading and orientation of satellite relative to the earth's magnetic field (second orbit)

knowing the curve in Fig. 13, it is possible to evaluate the parameters of the atmosphere or the aerodynamic parameters of the satellite, in agreement with Refs. 2 and 4.

6. Orientation Relative to the Sun

If ρ_0 and γ_0 are known, then the orientation of the vector of the kinetic moment relative to the sun can be computed in the following terms:

$$\begin{aligned} \cos\theta_{\odot} &= \cos\rho_0 \cos\kappa + \sin\rho_0 \sin\kappa \cos\mu^* \\ \cos\kappa &= \sin\Omega_{\odot} \cos I^* \quad 0 \leq \kappa \leq 180^\circ \\ \mu^* &= \gamma_0 - \tau_{\odot} \quad \sin\gamma_{\odot} = (\sin I^* / \sin\kappa) \sin\Omega_{\odot} \\ &\quad \cos\gamma_{\odot} = \cos\Omega_{\odot} / \sin\kappa \end{aligned}$$

Here θ_{\odot} is the angle between the vector of the kinetic moment and the direction to the sun; $I^* \approx 23^\circ.5$, the inclination of the equator to the ecliptic; Ω_{\odot} is the solar longitude from the vernal equinox (taken from *Astronomical Almanac*). The result of the calculation is shown in Fig. 14. It can be seen that θ_{\odot} increases from 30° on the first orbit to $\sim 90^\circ$ on the 100th orbit. Considering that the angle of nutation is close to 90° (tumbling of satellite) and taking into account the existence of characteristic rotation, the conclusion can be drawn that the satellite is illuminated (and heated) rather uniformly by the sun.

3. Computation of the Orientation of the Satellite Instruments in Space

The Eulerian angles φ, ψ, ϑ (Fig. 5) are of principal importance in the determination of the orientation of the satellite instruments. It is necessary to fix the position of axis x' in relation to the satellite in order to reckon angle φ . It was shown in Sec. 1 that it is convenient to select the start of transducer reading q_1 as axis x' . An analysis of transducer readings q_1 and q_2 permits determination of the start of the reading ("zero") q_1 , that is, the position of axis Ox' . The plane which passes through axis Ox' and Oz' forms an angle of 159° with the plane of disposition of the magnetic manometer (the angle is counted clockwise from the plane of the magnetic manometer to plane Ox').

The altitude of axis l of any device or any axis of the satellite relative to the system of coordinates x', y', z' , is determined by angles j and σ (Fig. 15), where j is the angle between axis l and z' of the satellite and angle σ is reckoned from x' and y' counterclockwise and is the angle between axis x' and projection l on plane $x'y'$.

The altitude of axis l is also given in terms of direction cosines m, n, k in relation to axis x', y', z' , respectively:

$$m = \sin j \cos \sigma \quad n = \sin j \sin \sigma \quad k = \cos j \quad [3.1]$$

1. Orientation Relative to the Earth's Magnetic Field

The altitude of the satellite relative to the earth's magnetic field can be determined completely. Actually, angle λ

between axis z' and the vector of magnetic intensity \mathbf{H} is given in terms of the graph $\lambda = q_2 - \frac{1}{3}q_1$, reckoned from a specific level which can be found on every orbit by analyzing the maxima of the readings; angle $\Delta \equiv q_1$ is the angle between the plane of reckoning angle λ and plane $z'x'$ (reckoned counterclockwise; Fig. 16). Thus, the altitude of the satellite relative to the vector of magnetic intensity is determined completely everywhere that there are readings of Δ and λ . Examples of these readings together with the level of reckoning angle λ are shown in Figs. 17 to 19 (where I is the reading of Δ , II the reading of q_2 , III the reading of λ , and IV the reading from which λ is reckoned).

If Δ and λ are known, then angle κ between \mathbf{H} and axis l of any device is simply computed:

$$\cos\kappa = \cos j \cdot \cos\lambda + \sin j \cdot \sin\lambda \cdot \cos(\Delta + \sigma) \quad [3.2]$$

Fig. 20 shows an example of computation in accordance with [3.2]: the angle between the axis of the photomultiplier ($j = 90^\circ, \sigma = 114^\circ$) and the vector of magnetic intensity on the second orbit. The readings of q_1 and q_2 on the second orbit show (Fig. 17) that the vector of the kinetic moment is close to the direction of the magnetic line of force (oscillations of λ are almost absent); that is, the angle between the satellite axis and vector \mathbf{H} is close to 90° . Therefore, angle κ (Fig. 16) does not exhibit clear, short-period oscillations which are brought about by the precession of the satellite.

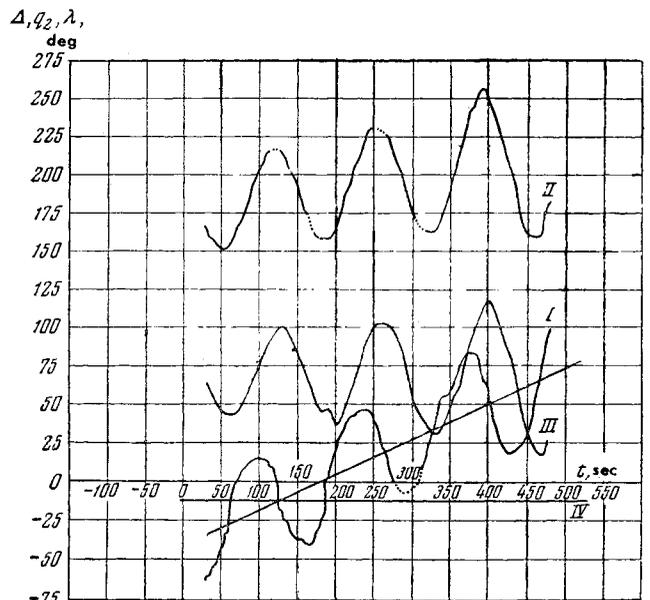


Fig. 18 Magnetometer reading and orientation of satellite relative to the earth's magnetic field (15th orbit). Sloping line—linear part of recording Δ

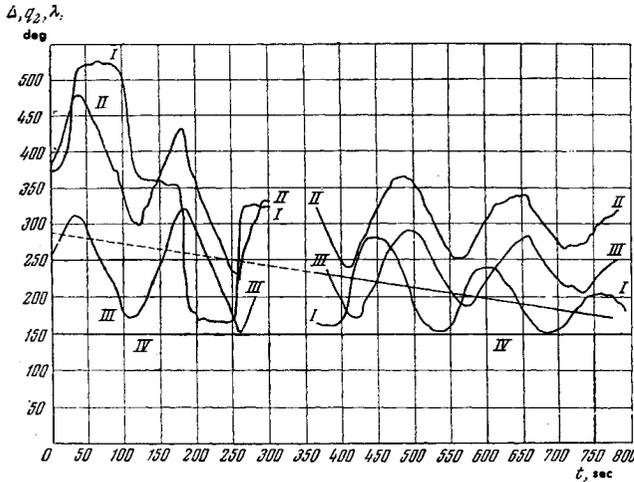


Fig. 19 Magnetometer reading and orientation of satellite relative to the earth's magnetic field (56th orbit). Sloping line—linear part of recording Δ

Long-period oscillations which are brought about by the rotation of the satellite around its own axis can be detected clearly.

2. Determination of the Orientation of the Fixed Axis of the Satellite Relative to the Fixed Direction in Space

Euler's angles

$$\begin{aligned} \varphi &= \varphi_0 + \dot{\varphi}t \\ \psi &= \psi_0 + \dot{\psi}t \\ \vartheta &= \vartheta_0 \end{aligned}$$

and the absolute coordinates ρ_0, γ_0 of the vector of the kinetic moment serve as the basis for the calculation. Parameters $\vartheta_0, \varphi, \psi, \rho_0, \gamma_0$ on every orbit are determined in the manner described in the foregoing sections.

The initial value φ_0 , in agreement with Eq. [1.7] is determined in accordance with the following:

$$\varphi_0 = \Delta(\lambda - \lambda_{\min}) \pm 90^\circ \quad [3.3]$$

or in accordance with its equivalent formula

$$\varphi_0 = \text{lin}\Delta \pm 90^\circ \quad [3.4]$$

where $\text{lin}\Delta$ is the linear part of the reading $\Delta \equiv q_1$. The equivalence of Eqs. [3.3] and [3.4] is readily verified directly on graphs q_1 and $\lambda \sim q_2 - \frac{1}{3}q_1$.

Parameter ψ_0 is related to parameter $\bar{\psi}_0$, employed in Secs. 1 and 2, in agreement with [1.5], through the following relationship:

$$\psi_0 = \bar{\psi}_0 - 90^\circ$$

The mutual attitude of systems X, Y, Z , and L, L_1, L_2 (Fig. 4) is determined from the table of direction cosines

	X	Y	Z
L_1	A_1	A_2	A_3
L_2	B_1	B_2	B_3
L_3	C_1	C_2	C_3
A_1	$= \cos\rho_0 \cos\gamma_0$		
B_1	$= \sin\gamma_0$		
C_1	$= \sin\rho_0 \cos\gamma_0$		
A_2	$= -\sin\rho_0$		
B_2	$= 0$		
C_2	$= \cos\rho_0$		
A_3	$= \cos\rho_0 \sin\gamma_0$		
B_3	$= -\cos\gamma_0$		
C_3	$= \sin\rho_0 \sin\gamma_0$		

The attitude of the principal axes x', y', z' of the satellite relative to system L_1, L_2, L_3 is determined by the direction cosines (7)

	x'	y'	z'
L_1	$\bar{\alpha}$	$\bar{\alpha}'$	$\bar{\alpha}''$
L_2	$\bar{\beta}$	$\bar{\beta}'$	$\bar{\beta}''$
L_3	$\bar{\gamma}$	$\bar{\gamma}'$	$\bar{\gamma}''$

$$\begin{aligned} \bar{\alpha} &= \cos\psi \cos\varphi - \sin\psi \sin\varphi \cos\vartheta \\ \bar{\alpha}' &= -\cos\varphi \sin\psi - \sin\psi \cos\varphi \cos\vartheta \\ \bar{\alpha}'' &= \sin\psi \cdot \sin\vartheta \\ \bar{\beta} &= \sin\psi \cos\varphi + \cos\psi \sin\varphi \cos\vartheta \\ \bar{\beta}' &= -\sin\psi \sin\varphi + \cos\psi \cos\varphi \cos\vartheta \\ \bar{\beta}'' &= -\sin\vartheta \cos\psi \\ \bar{\gamma} &= \sin\varphi \sin\vartheta \\ \bar{\gamma}' &= \cos\varphi \sin\vartheta \\ \bar{\gamma}'' &= \cos\vartheta \end{aligned}$$

The matrix product of these two tables of direction cosines determines the direction cosines of the axes of the satellite with respect to the inertial axes

	X	Y	Z
x'	α	β	γ
y'	α'	β'	γ'
z'	α''	β''	γ''

$$\begin{aligned} \alpha &= \bar{\alpha}A_1 + \bar{\beta}B_1 + \bar{\gamma}C_1 \\ \alpha' &= \bar{\alpha}'A_1 + \bar{\beta}'B_1 + \bar{\gamma}'C_1 \\ \alpha'' &= \bar{\alpha}''A_1 + \bar{\beta}''B_1 + \bar{\gamma}''C_1 \\ \beta &= \bar{\alpha}A_2 + \bar{\beta}B_2 + \bar{\gamma}C_2 \\ \beta' &= \bar{\alpha}'A_2 + \bar{\beta}'B_2 + \bar{\gamma}'C_2 \\ \beta'' &= \bar{\alpha}''A_2 + \bar{\beta}''B_2 + \bar{\gamma}''C_2 \\ \gamma &= \bar{\alpha}A_3 + \bar{\beta}B_3 + \bar{\gamma}C_3 \\ \gamma' &= \bar{\alpha}'A_3 + \bar{\beta}'B_3 + \bar{\gamma}'C_3 \\ \gamma'' &= \bar{\alpha}''A_3 + \bar{\beta}''B_3 + \bar{\gamma}''C_3 \end{aligned}$$

Finally, if m, n, k [3.1] are the direction cosines of fixed axis l in the satellite, then the direction cosines m_1, n_1, k_1 of the fixed axis l with respect to the inertial axes of coordinates X, Y, Z are determined as follows:

$$\begin{aligned} m_1 &= m\alpha + n\alpha' + k\alpha'' \\ n_1 &= m\beta + n\beta' + k\beta'' \\ k_1 &= m\gamma + n\gamma' + k\gamma'' \end{aligned}$$

Let \mathbf{V} be a fixed direction in space determined by direction cosines ν, μ, ξ in the system of inertial axes X, Y, Z . Then angle $\bar{\Theta}$ between fixed axis l and direction \mathbf{V} is determined by the following formulas:

$$\cos\bar{\Theta} = m_1\nu + n_1\mu + k_1\xi$$

For example, if \mathbf{V} is the direction of the velocity of the center of mass of the satellite, then

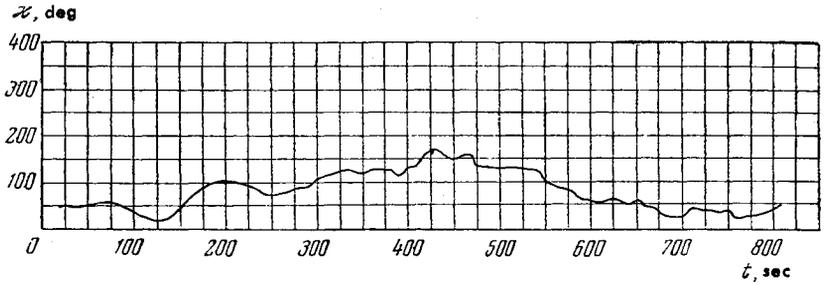
$$\nu = v_x/v \quad \mu = v_y/v \quad \xi = v_z/v$$

where v_x, v_y, v_z are velocity components along axes X, Y, Z , and $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

The quantities of m_1, n_1, k_1 , and angle $\bar{\Theta}$ between the fixed axis in the satellite and the velocity vector of the center of mass of the satellite were calculated in accordance with the specified formulas on an electronic computer.

Calculations show that the parameters obtained by processing the magnetometer readings must be corrected in order to bring the calculations into satisfactory agreement with the readings of the devices. Parameters φ_0 and ψ_0 were corrected within the limits of 30° . The inaccuracy in the determination of parameter ψ_0 was attributed to the deviation between the positions of the maxima determined by calculation and the maxima of the readings by 20-30 sec. The inaccuracy of

Fig. 20 Angle between the magnetic line of force and photomultiplier axis, according to the measurements made on the second orbit



parameter φ_0 is not so influential as that of parameter ψ and primarily leads to a variation of the size of the maxima. Other parameters ($\rho_0, \gamma_0, \varphi, \psi, \vartheta$) were hardly corrected at all (their variation has a very slight effect on the result of the calculation); the error in the determination of the magnitude of ψ , over a long period of time, results in an accumulation of error in the computation and a shift of the maxima. If such a shift took place in the case of a nominal ψ , then the quantity ψ was corrected. The magnitudes of ρ_0 and γ_0 were corrected only for these orbits where ρ_0 and γ_0 deviated from the system of values ρ_0 and γ_0 on other orbits.

The readings of the mass-spectrometer and the ionization manometer were used for the correction. Fig. 21 shows an example of the calculation of angle $\bar{\theta}$ between the manometer axis and the velocity vector of the center of mass of the satellite. Fig. 21 also shows the reading of a corresponding device for comparison.

Conclusion

This investigation shows that the readings of the magnetometer transducers housed on Soviet Sputnik III permit one to determine the parameters of rotation and orientation and, consequently, make it possible to calculate the orientation of any device in the satellite. The periods of precession and rotation of the satellite are determined within an accuracy of 5 sec; the angle between the satellite axis and the axis of precession which coincides with the direction of the vector of the kinetic moment, that is, the angle of nutation, is determined to an accuracy of 1° ; the attitude of the axis of precession in space is determined within an accuracy of 10° . These characteristics completely describe the nature and the evolution of satellite motion about the center of mass.

The determination of the initial time dependence of the satellite's attitude (initial values of the angles of the satellite's attitude (initial values of the angles of precession and rotation) involves a greater error (to 30°). Since the inaccuracy in the initial determination introduces only a constant error into the calculation, then the initial values of the parameters can be corrected by comparing the computed orientation with read-

ings of other devices. The computed orientation was in satisfactory agreement with the readings of the devices after such corrections were made.

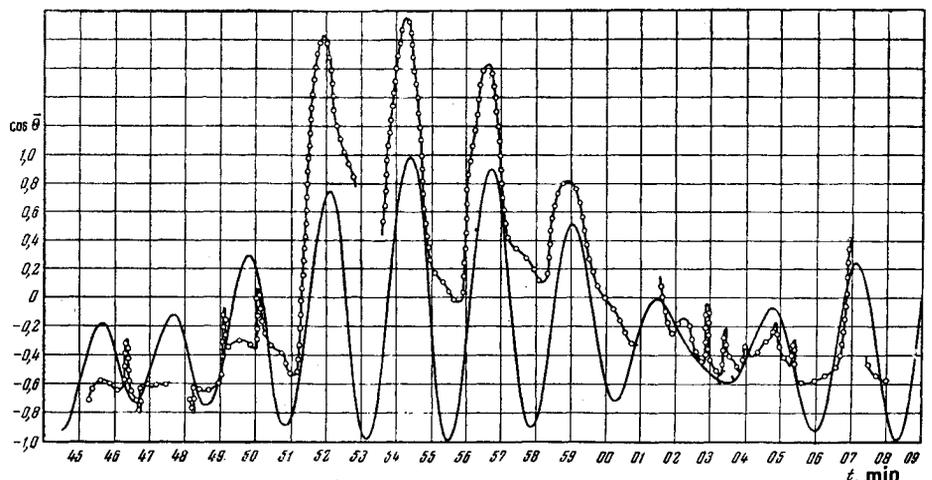
The "zeroes" of the magnetometer transducers were determined, and this made it possible to determine the orientation relative to the earth's magnetic field by a simple method directly from the readings of the magnetometer transducer.

The determined parameters of rotation and orientation of the satellite give an adequately clear picture of its motion about the center of mass for the first days of the satellite's life. It appears that during a period of time which is of the order of magnitude of the period of orbital rotation of the satellite, its motion about the center of mass can be considered an approximately regular precession. The parameters of regular precession change slowly from orbit to orbit.

The satellite rotates in a "tumbling" manner around the direction of the vector of the kinetic moment so that the angle between the satellite axis and the vector of the kinetic moment is close to 90° , deviating from this attitude not more than 6° on various orbits. The period of precession (the "tumbling" period) increases slowly from 135 to 140 sec on the first few (one to five) orbits and to 195 sec on the 283rd orbit. In addition, the satellite rotates slowly around its own symmetry axis. The angular velocity of this rotation decreases from $0.375 \text{ grad}\cdot\text{sec}^{-1}$ on the first orbit to zero on approximately the 20th orbit, after which the rotation direction reverses and the values of the angular velocity of the characteristic rotation on separate orbits vary around the average value of the order of $-0.1 \text{ grad}\cdot\text{sec}^{-1}$, deviating from this value not more than $0.1 \text{ grad}\cdot\text{sec}^{-1}$.

The vector of the kinetic moment, that is, the axis around which the satellite tumbles, changes its direction in space slowly at an angular velocity of the order of one deg/rotation of satellite along the orbit. The motion of the vector of the kinetic moment is such that for the investigated orbital range it tends toward the direction of the vector of the center of mass of the satellite at orbital perigee. Around the 100th to 110th orbit the vector of the kinetic moment almost coincides with the velocity vector at perigee so that the plane normal to the vector of the kinetic moment ("tumbling" plane of satellite)

Fig. 21 Orientation of the axis of the ionization manometer versus the velocity vector on the 15th orbit and comparison of this orientation with the manometer reading. Solid line—variation $\cos \bar{\theta}$, circled line—manometer reading in logarithmic scale



is also normal to the incident flow at orbital perigee. Consequently, on these orbits the satellite encounters maximum aerodynamic resistance. As a result of such a movement of the satellite in the perigee and in the apogee, its bottom part must face the earth periodically, this period being equal to the period of "tumbling."

A slow variation of the parameters of rotation and orientation of the satellite can be explained by the influence of perturbing factors: the effect of aerodynamic and gravitational perturbations (2,4), the interaction of the electrical circuits in the satellite with the earth's magnetic field, and, to a lesser degree, the effect of Foucault (5) currents, and other perturbing factors.

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Reviewer's Comment

The substance of this paper appears to be primarily the solution of a coordinate conversion problem done in some detail and probably quite well, but overlong by usual standards for publication in an American journal.

Information about the instrumentation by which the earth's magnetic field is instantaneously tracked, as well as the angular measuring techniques between different axis systems, would be of interest, but this is not included in the paper. It

is possible that some sleuthing would lead to characteristics of Sputnik III as revealed by the minor excursions in its orbit which appear to be disclosed in good faith.

None of us at MIT wishes to vouch for the detailed accuracy of the analysis, but the paper seems to be a high quality effort within the limited area of interest noted.

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Forms of Solutions of Einstein's Equations

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Obviously, there is altogether too great a variety of methods and additional assumptions, unconnected with the boundary conditions, now being used to solve Einstein's equations. This paper is an attempt to determine whether all these forms of solution are equivalent. It is found that some solutions contradict the basic postulates of tensor analysis when considered from the point of view of the two-metric theory of the gravitational field.^{1,2}

IN the general theory of relativity the gravitational field is regarded as an effect of the "deviation" of the real from the planar (Euclidian) space-time manifold. The latter is viewed as the physical abstraction of a space devoid of material sources. This fundamental idea is formulated with sufficient completeness in the so-called two-metric theory, which makes explicit use of the characteristics of two spaces, an auxiliary (planar) and a real (Riemannian) space. If $\hat{g}_{\mu\nu}$ is the metric tensor of a space without sources and $g_{\mu\nu}$ that of a space with sources, then the formation of a gravitational field must be viewed as the process of transformation (mapping) of $\hat{g}_{\mu\nu}$ into $g_{\mu\nu}$. Both tensors are essentially functions of the same coordinates; hence the properties and laws of transformation of the coordinates are not affected by the mapping.

In Einstein's ordinary theory, the characteristics of the starting ("empty") space play no part. In this connection, serious difficulties arise in interpreting the physical significance of certain geometrical objects, and this creates a fertile soil for speculations about the choice of a system of coordinates.³ If, however, we analyze the positive results of this theory, from the point of view of the two-metric formalism, in every case we can detect a tendency to realize the fore-mentioned basic idea.

Let us now consider a few of the methods used in solving gravitational equations.

In the so-called weak field approximation⁴ the metric tensor of the unknown space $g_{\mu\nu}$ is taken in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric (for a corresponding choice of coordinates), and $h_{\mu\nu}$ are small "corrections," approximately determined from the equations of gravitation. Here it is a question not only of comparing the unknown and starting (known) spaces but also of predetermining the properties of the starting space.

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